International Islamic University, Islamabad
Faculty of Basic and Applied Sciences
Department of Mathematics and Statistics

MS Admission Test
Max. Time: 90 Minutes

Note: Attempt as many questions as you can. Use separate Sheet for each question and write your name on every sheet.

Section-I

Q.No.1. *Advanced Calculus*.

a) Show that

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is $f_{xy} \neq f_{yx}$ at origin.

b) Use Green theorem to evaluate the line integral

$\oint 2y\,dx + 5x\,dy$ where C is the circle $(x-1)^2 + (y+3)^2 = 25$

Q.No.2. *Group Theory*. Let $\varphi : G \to G'$ be a group homomorphism. Then prove that

$$G/_{Ker\varphi} \cong \varphi(G).$$

Q.No.3. *Ordinary Diff. Eqs*. Find the eigenvalue and eigenfunction of a regular SLBVP

$$\frac{d}{dx}\left[\frac{1}{3x^2+1}\frac{dy}{dx}\right] + \lambda(3x^2+1)y = 0$$

Subject to the following boundary conditions

$$y(0) = 0,\ y(\pi) = 0$$

Q.No.4. *Diff. Geometry-I.* Find curvature of the following Curve:

$$\alpha(t) = (\cos t,\ \sin t,\ 0)$$

Q.No.5. *Complex Analysis.* Find the residue of the function $z\,Cos\left(\dfrac{1}{z}\right)$ at $z = 0$.

Q.No.6. *Linear Algebra.* Let $T : (R^3, R) \to (R^2, R)$ be defined by

$T(x_1, x_2, x_3) = (3x_1 + 2x_2 - x_3, x_1 - 4x_2 + 2x_3)$. Then show that T is a linear transformation and also the corresponding transformation matrix.

Q.No.7. ***Analytical Mechanics.***

(a) Determine whether

$$\mathbf{F} = \left(x^2 y - z^3\right)\mathbf{i} + \left(3xyz + xz^2\right)\mathbf{j} + \left(2x^2 yz + yz^4\right)\mathbf{k}$$

is conservative.

(b) Find moment of inertia of uniform circular disc of radius a about an axis through its centre and perpendicular to its plane.

Q.No.8. ***Topology.*** Let τ_1, τ_2 be two topologies on a non-empty set *X*. Then prove that $\tau_1 \cap \tau_2$ is always a topology on *X*. What about $\tau_1 \cup \tau_2$?. Give an example to support your answer.

Q.No.9. ***Real Analysis.*** The series $\sum x_n$ of real numbers converges if and only if for every $\in > 0$ there exists a positive integer $M(\in)$ such that $|s_m - s_n| = |x_{n+1} + x_{n+2} + ... + x_m| < \in$, whenever $m > n \geq M(\in)$.

Q.No.10. ***Partial Diff. Eqs.*** Find the solution of

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial^2 t} \qquad 0 < x < L, \quad 0 < t$$

$$u(0,t) = 0, \quad 0 < t$$

$$u(L,t) = 0, \quad 0 < t$$

$$u(x,0) = x, \quad 0 < x < L$$

Q.No.11. ***Functional Analysis-I.***

a) Prove that every normed space is a metric space.

b) Let X be a normed space and $T : X \to X$ be a linear operator. Then T is 1-1 if and only if $\ker(T) = \{0\}$.

Q.No.12. ***Numerical Analysis-I.*** Use interpolation to find f(2) and f'(2) given that f(0)=0, f(1)=1, f(3)=0.

-:Good Luck:-